

# Scattering Optimization Method for the Design of Compact Mode Converters for Waveguides

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**Abstract**—A method for the design of compact and efficient mode converters for waveguides is presented. The required mode converter is modeled in the form of a scatterer placed in the waveguide and then its surface profile is optimized for maximum excitation of power in the required mode. Design examples are provided which show that such converters can achieve efficiencies of above 98%, while keeping the length of the converter less than that of the conventional mode converters.

## I. INTRODUCTION

DESIGN of mode converters for overmoded waveguides has been of considerable interest since the development of high power, high frequency gyrotron oscillators for plasma heating. Various designs have been demonstrated for conversion from  $TE_{on}$  to  $TE_{om}$  modes in overmoded circular waveguides [1]–[3]. Apart from slight modifications in the perturbation profile, all these designs are primarily based on the technique developed by Kovalev *et al.* [4], utilizing coupled wave theory [5]. In this technique, the walls of the waveguide are slightly perturbed periodically to develop a structure similar to a diffraction grating. The period of the perturbation,  $\delta$ , is approximately determined by the difference in the propagation constants of the input and the output modes [2], i.e.,

$$\delta = 2\pi/|\beta_m - \beta_n|. \quad (1)$$

This periodic structure converts a waveguide mode  $TE_{on}$  to  $TE_{om}$  in much the same way as a diffraction grating converts a free space mode into another. Conversion efficiencies ranging from 95–99.5% have been reported for such converters [1]–[3]. However, these mode converters tend to be physically long. Despite various efforts [3], [6]–[9], the length of converters designed using this approach remains large compared to the radial dimensions of the waveguide. For example, for a circular waveguide with radius  $a = 1.389$  cm and a frequency of 150 GHz, Buckley and Vernon [3] report a length of 57.1 cm for a  $TE_{02}$  to  $TE_{01}$  converter, with a calculated efficiency of 99.5%. For the same waveguide and frequency, Thumm and Kumric [7] report a length of 107.5 cm for a  $TE_{03}$  to  $TE_{01}$  mode converter, with a predicted efficiency of 97.8%.

To resolve the problem of length and also to explore other possibilities, a different approach to the design of mode

converters for waveguides is proposed. Our method is based on a short forceful impact rather than a periodic field perturbation caused by a grating like structure. For designing a mode converter, the problem is formulated as that of finding a scattering surface (formed by varying the transverse dimensions of the waveguide) which can scatter the incoming field in such a way as to convert it from one mode structure to the other. Such a scatterer is modeled in the waveguide as a composition of a finite number of discontinuities in the waveguide walls. The mode matching method is used to analyze these discontinuities [10]–[12]. Mode matching enables the development of a generalized scattering matrix which can be used to find the power scattered into each mode at the input and output planes of the scatterer. By changing the surface profile of the scatterer, it is then possible to control the power in the mode of interest at the output. The shape of the required mode converter is then found by optimizing the power in the required mode as a function of the surface profile of the scatterer.

In the following sections, the scattering optimization method for mode converter design is introduced by implementing it for parallel plate waveguides. A parallel plate geometry is selected for this initial implementation for conceptual and numerical simplicity. Some examples of mode converters designed using this technique are presented and it is shown that these converters achieve predicted efficiencies of above 98%, while keeping the overall length of the transducer within an order of magnitude of the transverse waveguide dimension (height of the parallel plate waveguide).

## II. PROBLEM FORMULATION

The aim is to design a mode converter in the form of a scattering surface which would transform a set of input modes in such a way as to concentrate power into a required mode at the output. Consider a set of modes incident from the left or right side in a parallel plate waveguide. Varying the height of the waveguide would result in scattering of the incident energy into various modes. If the incident mode is a TE mode, all the energy will be scattered into TE modes, similarly for TM modes. Consider an arbitrary scatterer realized by varying the height of the waveguide as shown in Fig. 1(a). In the design approach followed in this paper, we attempt to find the optimum scatterer surface for maximum conversion of power into the required output mode. The scheme for the search of this optimum mode converter surface is developed in the next few sections.

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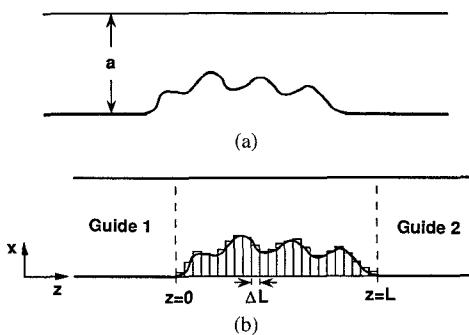


Fig. 1. (a) An arbitrary scatterer in a parallel plate waveguide. (b) Staircase model for an arbitrary scatterer.

### III. MODELING AN ARBITRARY SCATTERER

First of all, a technique is developed for modeling and analyzing an arbitrary scatterer in a parallel plate waveguide, as shown in Fig. 1. The aim is to determine the mode structure of the field pattern at the input plane ( $z = 0$  or  $z = L$ ) and the exit plane ( $z = L$  or  $z = 0$ ), when a field with an arbitrary mode content strikes the scatterer from the left or the right side. To do this, the scatterer surface is approximated in the form of a staircase, as in Fig. 1(b), by dividing its length into very thin sections and keeping the height constant for the length of each section. The scatterer has now been modeled in terms of a finite number of discontinuities in the height of the waveguide. Each of these discontinuities can then be analyzed numerically by using mode matching. Mode matching equations enable the development of a generalized scattering matrix for each step/junction of the staircase model. Using the method of Itoh and Chu [13], these generalized matrices for individual steps can be cascaded to get a unified generalized matrix for the complete structure of the scatterer.

Consider an arbitrary field pattern incident at any or both of the planes  $z = 0$  and  $z = L$ . The transverse fields in the presence of the scatterer at each cross-section can be expressed as a superposition of an infinite number of modes as follows:

At  $z = 0$

$$\bar{E}_T^{(1)} = \sum_{i=1}^{\infty} a_i^{(1)} \bar{e}_i^{(1)} + \sum_{i=1}^{\infty} b_i^{(1)} \bar{e}_i^{(1)} \quad (2)$$

$$\bar{H}_T^{(1)} = \sum_{i=1}^{\infty} a_i^{(1)} Y_i^{(1)} \bar{h}_i^{(1)} - \sum_{i=1}^{\infty} b_i^{(1)} Y_i^{(1)} \bar{h}_i^{(1)} \quad (3)$$

At  $z = L$

$$\bar{E}_T^{(2)} = \sum_{i=1}^{\infty} a_i^{(2)} \bar{e}_i^{(2)} + \sum_{i=1}^{\infty} b_i^{(2)} \bar{e}_i^{(2)} \quad (4)$$

$$\bar{H}_T^{(2)} = \sum_{i=1}^{\infty} a_i^{(2)} Y_i^{(2)} \bar{h}_i^{(2)} - \sum_{i=1}^{\infty} b_i^{(2)} Y_i^{(2)} \bar{h}_i^{(2)} \quad (5)$$

where  $\bar{e}_i^{(1,2)}$  and  $\bar{h}_i^{(1,2)}$  are the mode functions for the electric and magnetic fields, for the  $i$ th mode in waveguides 1 and 2,  $a_i^{(1,2)}$  and  $b_i^{(1,2)}$  are the complex incident and reflected mode coefficients, for the  $i$ th mode in waveguides 1 and 2,

and  $Y_i^{(1,2)}$  are the wave admittances for the  $i$ th mode, in waveguides 1 and 2, respectively. The mode functions are normalized so that the power in each mode is proportional to the magnitude squared of the related mode coefficient. To relate the unknown  $b_i$  with the known  $a_i$  in (2)–(5), we invoke the continuity of transverse electric and magnetic fields at each step discontinuity and the vanishing of the tangential electric field on the perfectly conducting obstacle (waveguide walls). Following Carin [14], a generalized scattering matrix  $S_i$  for the  $i$ th staircase step is developed that can handle an arbitrary number of propagating and evanescent modes incident from both sides of the discontinuity. For fastest convergence, the boundary enlargement and reduction cases are dealt with separately, as described by Wexler [10] and Carin [14]. For each uniform section between steps, a transmission matrix  $T_i$  is formed. The generalized scattering matrices  $S_1, \dots, S_k$  for the  $k$  discontinuities and the relevant transmission matrices [13] are then combined to form a composite scattering matrix,  $S$ , representing the effect of the whole length of the scatterer. The matrix  $S$  then relates the mode coefficients  $b_i$  with  $a_i$  in the following way:

$$B = SA, \quad (6)$$

where

$$B = \begin{bmatrix} b_1^{(1)} \\ \vdots \\ b_N^{(1)} \\ b_1^{(2)} \\ \vdots \\ b_N^{(2)} \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} a_1^{(1)} \\ \vdots \\ a_N^{(1)} \\ a_1^{(2)} \\ \vdots \\ a_N^{(2)} \end{bmatrix}.$$

In this paper only the case of TE modes incident from the left (guide 1) is considered. Thus, all  $a_i^{(2)}$  are taken to be zero and the calculations include mode functions for TE modes only. As is evident in (6), we have truncated the system of infinite equations (2)–(5) to a total of  $2N$  equations.  $N$ , the total number of modes (propagating and evanescent) is selected such that the mode matching solution converges [12], [15]. The system of equations given by (6) is now in a convenient form to allow the determination of the mode coefficients  $b_i$  and, thus, the mode pattern of the field at the input and exit planes ( $z = 0$  and  $z = L$ ). Using the mode coefficients  $b_i^{(2)}$  and  $b_i^{(1)}$  we can also find the power transmitted and reflected in each mode by the scatterer.

### IV. STUDY OF OPTIMIZATION SURFACE AND DISCRETIZATION STEP SIZE

The previous section introduced the technique used to find the power excited in each mode by a scatterer at its exit plane. For the purpose of designing a mode converter, we identify a general scatterer by a set of variables which are the lengths and heights (measured from  $x = 0$  in Fig. 1(b)) of the steps in the staircase scatterer model. The power excited by this scatterer in a particular mode is a function of its profile, specified by the lengths and heights of the staircase steps. If the length of each

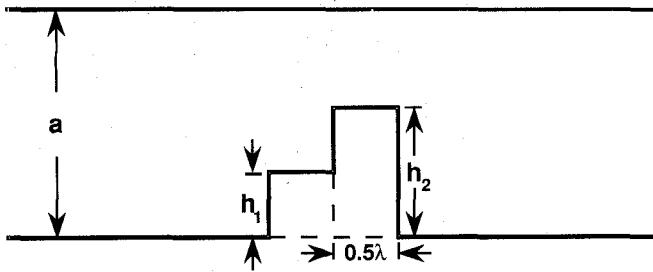
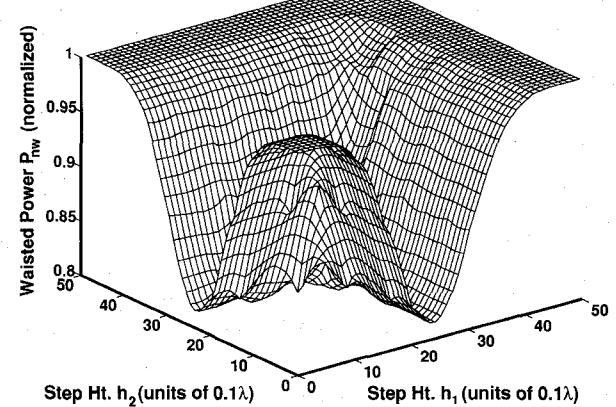
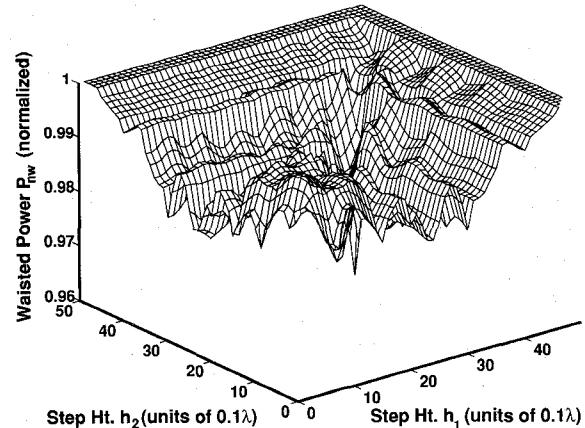


Fig. 2. A two-step scatterer.

step is kept constant, then the power excited in the  $n$ th mode at the exit plane of the scatterer can be written as a function of the step heights,  $\text{Power in mode } n = P_n(h_1, h_2, \dots, h_k)$ , where  $h_i$  is the height of the  $i$ th step. Maximizing  $P_n(h_1, h_2, \dots, h_k)$  would then amount to finding the scattering surface that would result in maximum conversion of power to the required mode  $n$ . Maximizing  $P_n$  is the same as minimizing the waisted power  $P_{nw}$  ( $P_{nw} = 1 - P_n$ ), where powers are normalized to 1. Thus, the design of the required mode converter has now been reduced to optimizing a function of  $k$  variables, which are the heights of steps in the scatterer.

To develop an optimizing scheme for  $P_{nw}$ , the smoothness of the surface generated in the  $k$ -dimensional space by the function  $P_{nw}$  is first explored. This is done by considering a sample case of a two step scatterer (heights  $h_1$  and  $h_2$ ), as shown in Fig. 2. At a frequency of 10 GHz, a TE<sub>1</sub> mode is considered incident at  $z = 0$  on the scatterer placed in a waveguide with dimension  $a = 15$  cm ( $5\lambda$ ), where  $\lambda$  is the free space wavelength. The length of each step in the scatterer is taken to be  $0.5\lambda$ . Heights  $h_1$  and  $h_2$  are varied and the output power  $P_n$  in mode TE <sub>$n$</sub>  at  $z = L$  is calculated for combinations of  $h_1$  and  $h_2$ .  $P_{nw}$  is found from  $P_n$  and is plotted versus  $h_1$  and  $h_2$  for  $n = 2$  and 6 in Figs. 3 and 4. It is noticed that as the exit mode number is increased farther away from the incident mode number, the optimization surface becomes more and more complex. It was also observed that as the wave guide size is increased, the plotted surfaces become more complicated even for lower order exit modes. It is therefore concluded that a local optimization technique might work satisfactorily for conversion to adjacent modes in a small size waveguide, but for highly overmoded waveguides a sophisticated global optimization technique is required. This makes the problem difficult, as we are left with finding a global optimum in a  $k$  dimensional space. The main limitation of any global optimization algorithm is that one is never certain if the global minimum has been reached, especially when dealing with a multi-dimensional space. Fortunately, in our problem the ideal global minimum value that the function of interest may achieve is known, i.e.,  $P_{nw} = 0$ .

The shape of the staircase model of Section III depends upon the step length ( $\Delta L$ ) and height increments chosen to model a scatterer. Ideally, the step length should be very near to zero, but that would mean  $k \rightarrow \infty$  and the optimization problem would become unsolvable. A very small  $\Delta L$  will also not be feasible from a fabrication standpoint.  $\Delta L$  is therefore restricted to  $0.1\lambda$  or more. Step height increments, however, must be established by considering performance sensitivity, as discussed in Section VI.

Fig. 3. Optimization surface for TE<sub>1</sub> to TE<sub>2</sub> conversion.Fig. 4. Optimization surface for TE<sub>1</sub> to TE<sub>6</sub> conversion.

## V. OPTIMIZATION PROBLEM

This section deals with the primary issue of optimizing the scattering surface for maximum conversion of power to the required mode. If the scatterer is divided into steps of  $0.1\lambda$ , the number of variables for optimization may become very large, resulting in a complicated optimization surface. This surface can be extremely rough with many local valleys similar to those in Fig. 4. In the search for global minimum in this high dimensionality space, a solution constitutes a local minimum which gives the function value acceptably close to the ideal value,  $P_{nw} = 0$ . One of the key issues is the selection of an adequate scatterer length. The mode converter length needed will vary depending upon the size of the waveguide, frequency of operation, and the number of propagating modes. In our approach, the mode converter length is specified in terms of the number of variables (step heights),  $k$ , used for optimization, once  $\Delta L$  is chosen. This establishes the degrees of freedom available for the converter design. The conversion efficiency is defined as  $\eta = 100(P_n/P_{in})$ , where  $P_{in}$  is the total incident power. The efficiency should increase as  $k$  is increased and may approach 100% as  $k \rightarrow \infty$ . To ensure convergence to an acceptable local minimum, a dynamic optimization concept is invoked, in which the optimization surface is re-structured at every stage of the solution procedure so as to enable

easy access to the required valley. The re-structuring of the surface is carried out by increasing the number of variables,  $k$ , at every stage, which implies increasing the length of the scatterer (assuming a fixed step length). Thus, in our method we start with an arbitrary number of variables and carry out the optimization while adding more variables until an acceptable efficiency is reached.

Searching for minima in a large multi-dimensional surface of complex shape is computationally unrealistic unless the optimization sequence is started with a guess very near to the actual minimum. To achieve a tractable solution a hierarchical approach to optimization is implemented as follows. Let the minimum step length ( $\Delta L$ ) in our design be  $0.1l$ , where  $l \approx \lambda$ , the free space wavelength. First of all, a coarse optimization is carried out by modeling the scatterer with steps of length  $l$ , thus reducing the number of variables by a factor of 10. This gives a coarse estimate for the optimum mode converter surface, which is further subdivided into steps of length  $0.5l$  and the optimization carried out again. This refinement of length is further extended to steps of length  $0.1l$  and the final optimization sequence is carried out.

Along with the hierarchical optimization sequence, another refinement sequence is introduced which is motivated by the amount of processing time required by the mode matching solution. Mode matching involves the solution of a linear matrix equation of the form of (6) at each step junction and extensive matrix multiplications and inversions for cascading the scattering matrices for individual steps. The overall processing time for these calculations increases exponentially with the number of total modes used,  $N$ .  $N$  has to be large, with the inclusion of evanescent modes, to achieve convergence to an accurate solution. To resolve this issue, the whole optimization sequence (as described above) is initially carried out by considering none or very few evanescent modes in the mode matching calculation. Subsequently, a suitable number of evanescent modes is introduced in the last refinement stage of the optimization sequence.

Global optimization techniques converge best when the number of variables for optimization is small, whereas the number of variables (heights) for modeling a scatterer can range in the hundreds. To resolve this issue, a sectional approach to optimization is followed in which  $m$  heights are optimized at a time, while keeping the rest constant. We start with the first  $m$  heights and move across the scatterer length in increments of  $m$ , and then return to the beginning. This sequence is repeated until no further improvement can be achieved. This sequential sectional optimization scheme is found to work well for the type of optimization surfaces encountered in mode converter design. One limitation however is the large number of function evaluations (mode matching solution for power in the desired mode) required for optimization. Time spent on function evaluations, which is bulk of the total computation time, is reduced in the following way. Consider  $k$  scatterer heights, of which  $m$  are optimized at a time, while keeping the others constant. Three generalized scattering matrices,  $S^1$ ,  $S^2$ , and  $S^3$ , are developed:  $S^1$  for the steps before the  $m$  steps,  $S^2$  for the  $m$  steps, and  $S^3$  for the steps after the  $m$  steps. When iterating to optimize the  $m$

steps,  $S^1$  and  $S^3$  are not changed; only  $S^2$  is calculated for each new value of the  $m$  step heights.  $S^1$ ,  $S^2$ , and  $S^3$  are then combined to form the composite generalized scattering matrix,  $S$ . Avoiding the calculation of  $S^1$  and  $S^3$  in the optimization sequence considerably reduces the processing time.

Based on the above discussion, we can now enumerate the specific stages of designing a mode converter using the scattering optimization technique:

- 1) First assume an initial length,  $L$ , for the converter and a starting guess for its shape (vector comprising heights, given a step length). Choose an arbitrary converter length in multiples of  $l$ , where  $l \approx \lambda$ . This length should be much smaller than the desired mode converter length, say  $2a$  or  $3a$ , where  $a$  is the waveguide height. A reasonable initial guess for the converter surface would be a scatterer that gradually increases in height and is able to considerably disturb the incoming field. Say the length  $L$  of this guess is  $k_1 l$ . This means we have  $k_1$  steps in the initial guess, each of length  $l$ . It is found (as would be clear from examples in Section VI) that the mode converters based on this technique generally consist of a major scatterer in the center, preceded by and followed by some tuning elements, as shown in Fig. 5.
- 2) Maximize power in the required mode  $P_n$  by optimizing the  $k_1$  variables in the initial guess. After an optimum is achieved, gradually start adding steps (tuning elements) of length  $l$  at the beginning and end of the converter. Every time an element is added, the optimization is re-run to maximize  $P_n$ . Stop adding further elements when no considerable improvement in power results from an increase in the length of the scatterer. Say at this point the length of the converter is  $k_2 l$ . We now have a coarse estimate for the surface of our optimum mode converter.
- 3) Divide the coarse surface achieved after Stage 2 into steps of  $0.5l$  each, so that the number of variables now is increased to  $2k_2$ . An optimization sequence as in Stage 2 is again carried out and tuning elements are added on both sides if required. Stop the sequence when no noticeable improvement occurs in  $P_n$  with a further increase in the converter length. The length of the converter at this stage, say, is  $k_3(0.5l)$ .
- 4) The structure achieved after Stage 3 is divided into steps of  $0.1l$ , making the total number of variables for optimization  $5k_3$ . The optimization sequence of Stage 2 is again repeated and further tuning elements are added if required. The length  $L$  now becomes  $k_4(0.1l)$ .
- 5) Finally, a suitable number of evanescent modes are included in the calculations and Stage 4 is repeated.

The structure achieved after Stage 5 is then the optimized mode converter. The decision when to stop adding further elements in Stages 2 and 3 is important. To restrict the length of the converter, it would be best to add as few elements at these stages as possible before reaching Stage 4, where the necessary refinement can be done. It must be kept in mind that Stages 2 and 3 in the design procedure are there only to allow the development of an initial estimate for Stage 4. However, occasionally we may be able to find a reasonable solution

during one of these stages. In such a situation, one would skip over to Stage 5 directly and find the refined solution.

## VI. DESIGN EXAMPLES

Four of the mode converters designed using the above technique are presented in Table I. Structures achieved for the first two of these mode converters are shown in Figs. 5 and 6. In all these designs the IMSL routine BCPOL is used for global optimization. This routine works satisfactorily for our problem, provided the number of variables for optimization,  $m$ , is limited to two. A total of twenty evanescent TE modes are considered in each example to ensure convergence of the mode matching solution in Stage 5. The salient features of the mode converters so designed are summarized below:

- 1) The general shape of these mode converters comprises a primary scatterer in the center, flanked on both sides by smaller scatterers that act as tuning elements. The optimization sequence thus serves to tune these smaller elements to reduce reflection and maximize power in the required mode, much the same way as the stages of a filter are tuned. The heights of the tuning elements are very critical and can greatly affect the mode pattern at the output of the converter. Fig. 7 shows the shape of the TE<sub>1</sub> to TE<sub>2</sub> mode converter of Fig. 5 at various stages of design. The powers achieved at every step of solution refinement are given for comparison. The thing to note here is how the finer detail of the structure is picked up as the refinement in step length approaches  $0.1\lambda$  and the height elements are tuned to get the maximum power conversion.
- 2) The efficiency achieved in each of the designs is  $>98\%$ , which is considered good for a mode converter [1]–[3], [6]–[9].
- 3) The length of the converter in each case is not very large in comparison to the waveguide height. The lengths are much smaller than those achievable through the conventional coupled mode design of converters. The length of a conventional mode converter is approximately equal to one or more periods of the beat wavelength,  $\delta$ , given by (1). For conversion into a mode farther away from the incident mode, a larger number of beat periods are required. In Table I, the beat wavelength for each of the mode converters is given for comparison.
- 4) The mode conversion structures obtained through this method are not unique, because more than one local minimum exists which has a function value very close to the global minimum. Fig. 8 shows three structures for TE<sub>1</sub> to TE<sub>2</sub> mode conversion. Their conversion efficiencies are 99.47%, 99.8% and 99.77%, respectively, which are very close to the ideal maximum value of 100%. Note that the shapes of these three structures differ considerably.
- 5) Large processing times are required to design these mode converters, e.g., the CPU time for developing Design 1 on an Ardent Titan-P3 was nearly 4.5 hours.

One way of fabricating these mode converters for parallel plate waveguides is by stacking a series of plates inside

TABLE I  
DATA ON VARIOUS MODE CONVERTER DESIGNS

Converter Type	Frequency (GHz)	'a' (cm)	Efficiency (%)	Length of Converter	Beat Wavelength
TE <sub>1</sub> -TE <sub>2</sub>	3.5	15	99.47	45.58 cm	62.6 cm
TE <sub>1</sub> -TE <sub>4</sub>	4.5	15	98.62	58.96 cm	12.9 cm
TE <sub>1</sub> -TE <sub>2</sub>	10.0	15	98.56	111.9 cm	197.4 cm
TE <sub>3</sub> -TE <sub>1</sub>	10.0	15	98.31	151.5 cm	73.08 cm

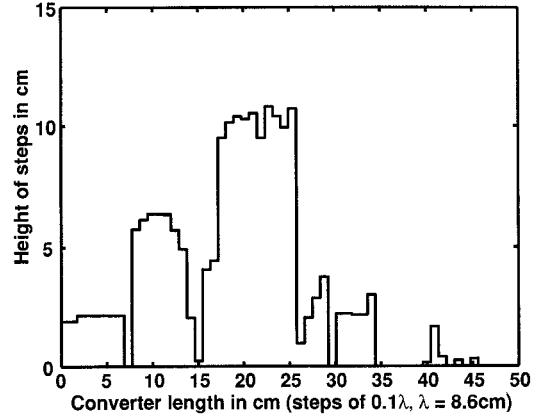


Fig. 5. TE<sub>1</sub> to TE<sub>2</sub> mode converter at 3.5 GHz in a waveguide with  $a = 15$  cms.

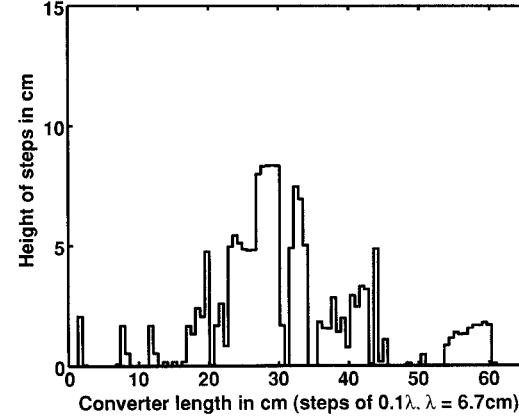


Fig. 6. TE<sub>1</sub> to TE<sub>4</sub> mode converter at 4.5 GHz in a waveguide with  $a = 15$  cms.

the lower wall of the waveguide. For circular waveguides the construction would be simpler through the use of thin metallic washers. Consider the effect of fabrication errors on the performance of these mode converters and their operating frequency bandwidth. How the step height fabrication errors affect the conversion efficiency depends on the size, shape and smoothness of the valley in the optimization surface, where the calculated minimum function value is located. No general rules for the effect of such errors can be identified, because variation in every step height has a different effect on efficiency. An important parameter in the mode converter design is the height increment used while carrying out the optimization sequence. The height increment determines the coarseness of the grid used for optimization. A coarser grid implies that most likely the solution lies in a larger valley, with a larger radius of curvature and therefore is not very sensitive

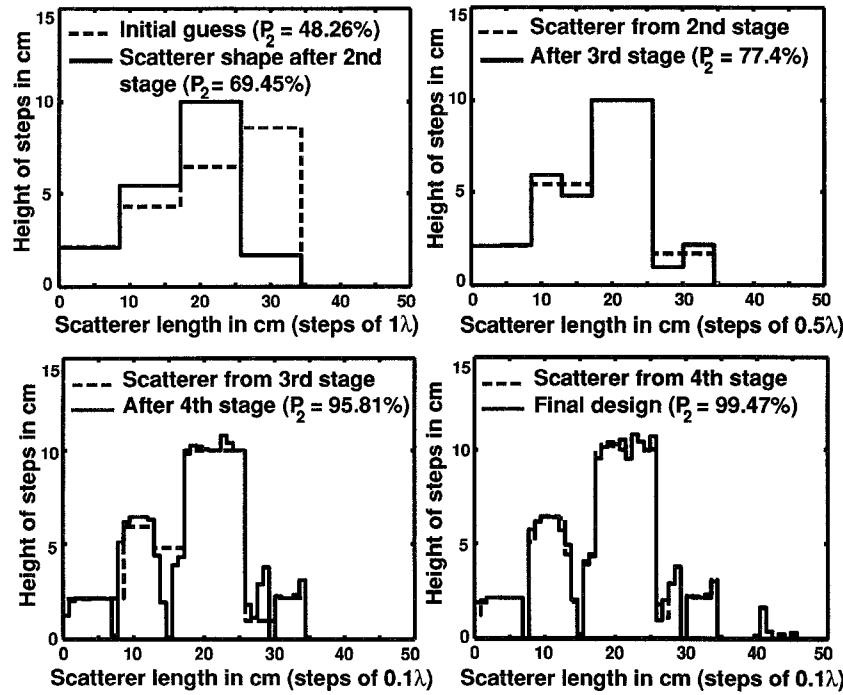


Fig. 7. Shape of the mode converter of Fig. 5 at various steps in the design procedure.

to small changes in element height. Consider a simple example wherein three mode converters are designed for  $TE_1$  to  $TE_2$  mode conversion at 3.5 GHz. These are shown in Fig. 8. In Design 1, the increments in height are allowed to be in steps of 0.1 mm during the optimization sequence. In Design 2, heights are varied with steps of 2 mm, while in Design 3 the variation in heights is in steps of 3 mm. It is very likely that the minimum achieved in the third design lies in a wider valley, because of the coarse grid, while in Design 1 the valley may be narrower. Consider how uniformly distributed random variations in step heights affect these designs. In Design 1, the decrease in efficiency remains within 1% provided the variations in heights is within  $\pm 0.07$  mm. For 1% change in efficiency this limit for Design 2 is  $\pm 0.3$  mm. In Design 3, a uniformly random variation of heights between  $\pm 0.5$  mm produces a maximum change of 0.7% in conversion efficiency. This shows that indeed the solution of Design 3 lies in a wider valley with a larger average radius of curvature. Changes in step lengths do not affect the efficiency as critically as the step heights, e.g., in Design 3, a  $\pm 0.5$  mm uniformly random variation in lengths only causes a 0.3% reduction in efficiency. In the same design, a random variation of  $\pm 0.5$  mm in both step heights and step lengths does not reduce efficiency beyond 0.8%. The bandwidth around the center frequency for which the efficiency remains within 1% is 2 MHz for Design 1, 12 MHz for Design 2 and 14 MHz for Design 3. Beyond these bandwidth limits, the efficiency decreases very sharply. Notice that the design which is less sensitive to height variations is also less sensitive to frequency variations.

The above-mentioned examples show that it is possible in a confined structure, like a parallel plate waveguide, to convert power from one mode to another using structures smaller in length than those based on the conventional periodic pertur-

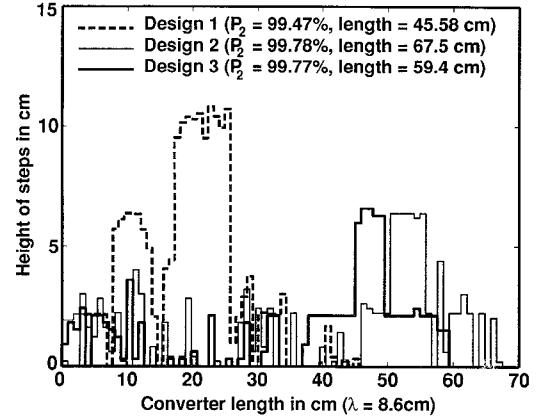


Fig. 8. Three designs for  $TE_1$  to  $TE_2$  mode converter at 3.5 GHz in a waveguide with  $a = 15$  cms.

bation technique. This result can have interesting implications in various fields in which periodic field excitation is used to modify a field pattern. Examples are the design of directional antennas and that of compact structures in optics which may behave in the same way as a larger size diffraction grating. In regard to waveguide mode converters, our method of design provides an added advantage. This method allows the design of mode converters to convert energy from a set of modes at the input to a single mode at the output, provided the mode coefficients for all the incident modes are known.

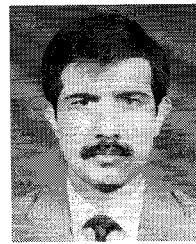
## VII. CONCLUSION

We have developed a new technique for the design of mode converters in over-moded waveguides, wherein the profile of a scattering surface is optimized for maximum conversion of power into the required mode at the output. Converters

designed using this approach are highly efficient and their overall length is smaller than the usual mode converters based on periodic wall perturbations. The scattering optimization technique demonstrated for parallel plate waveguides can be adopted for design of mode converters for other waveguide geometries. The general shape of the mode converters designed using this method consists of a central major scatterer with tuning elements on both ends. The heights of these tuning elements are critical in determining the efficiency of conversion. This technique not only reduces the length of the converters, but it also provides the advantage that more than one mode at the input can be converted into a single mode at the output. A further study of the results presented in this paper could impact the design of compact (leaky) optical waveguide diffractive surfaces.

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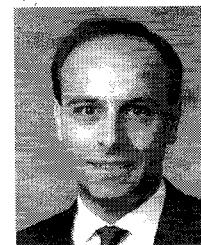
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